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Sliding Mode Control with Adaptive Fuzzy Dead-Zone Compensation of an Electro-hydraulic Servo-System

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Abstract Electro-hydraulic servo-systems are widely employed in industrial applications such as robotic manipulators, active suspensions, precision machine tools and aerospace systems. They provide many advantages over electric motors, including high force to weight ratio, fast response time and compact size. However, precise control of electro-hydraulic systems, due to their inherent nonlinear characteristics, cannot be easily obtained with conventional linear controllers. Most flow control valves can also exhibit some hard nonlinearities such as dead-zone due to valve spool overlap. This work describes the development of an adaptive fuzzy sliding mode controller for an electro-hydraulic system with unknown dead-zone. The boundedness and convergence properties of the closed-loop signals are proven using Lyapunov stability theory and Barbalat's lemma. Numerical results are presented in order to demonstrate the control system performance.

Keywords Adaptive algorithms · Dead-zone · Electro-hydraulic systems · Fuzzy logic · Nonlinear control · Sliding modes

Mathematics Subject Classifications (2000) 34H05 · 70Q05 · 93B12 · 93C10 · 93C15 · 93C95 · 93D05

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1 Introduction

Electro-hydraulic actuators play an essential role in several branches of industrial activity and are frequently the most suitable choice for systems that require large forces at high speeds. Their application scope ranges from robotic manipulators to aerospace systems. Another great advantage of hydraulic systems is the ability to keep up the load capacity, which in the case of electric actuators is limited due to excessive heat generation.

However, the dynamic behavior of electro-hydraulic systems is highly nonlinear, which in fact makes the design of controllers for such systems a challenge for the conventional and well established linear control methodologies. The increasing number of works dealing with control approaches based on modern techniques shows the great interest of the engineering community, both in academia and industry, in this particular field. The most common approaches are the adaptive [1–4] and variable structure [5–7] methodologies, but nonlinear controllers based on quantitative feedback theory [8], optimal tuning PID [9], adaptive neural network [10] and adaptive fuzzy system [11] were also presented.

In addition to the common nonlinearities that originate from the compressibility of the hydraulic fluid and valve flow-pressure properties, most electro-hydraulic systems are also subjected to hard nonlinearities such as dead-zone due to valve spool overlap. It is well-known that the presence of a dead-zone can lead to performance degradation of the controller and limit cycles or even instability in the closed-loop system. To overcome the negative effects of the dead-zone nonlinearity, many works [12–17] use an inverse function even though this approach leads to a discontinuous control law and requires instantaneous switching, which in practice can not be accomplished with mechanical actuators. An alternative scheme, without using the dead-zone inverse, was originally proposed by Lewis et al. [18] and also adopted by Wang et al. [19]. In both works, the dead-zone is treated as a combination of a linear and a saturation function. This approach was further extended by Ibrir et al. [20] and by Zhang and Ge [21], in order to accommodate non-symmetric and unknown dead-zones, respectively.

In this work, an adaptive fuzzy sliding mode controller is developed for an electro-hydraulic system subject to an unknown dead-zone input. The adopted approach is based on a control scheme recently proposed by Bessa et al. [22] for nth-order uncertain nonlinear systems, that does not require previous knowledge of dead-zone parameters nor the construction of an inverse function. On this basis, a smooth sliding mode controller is considered to confer robustness against modeling imprecisions and a fuzzy inference system is embedded in the boundary layer to cope with dead-zone effects. The boundedness and convergence properties of the closed-loop system are analytically proven using Lyapunov stability theory and Barbalat's lemma. Numerical simulations are carried out in order to demonstrate the control system performance.

2 Electro-hydraulic System Model

In order to design the adaptive fuzzy controller, a mathematical model that represents the hydraulic system dynamics is needed. Dynamic models for such systems are well documented in the literature [23, 24].

The electro-hydraulic system considered in this work consists of a four-way proportional valve, a hydraulic cylinder and variable load force. The variable load force is represented by a mass–spring–damper system. The schematic diagram of the system under study is presented in Fig. 1.

The balance of forces on the piston leads to the following equation of motion:

$$F_g = A_1 P_1 - A_2 P_2 = M_t \ddot{x} + B_t \dot{x} + K_s x \tag{1}$$

where F_g is the force generated by the piston, P_1 and P_2 are the pressures at each side of cylinder chamber, A_1 and A_2 are the ram areas of the two chambers, M_t is the total mass of piston and load referred to piston, B_t is the viscous damping coefficient of piston and load, K_s is the load spring constant and x is the piston displacement.

Defining the pressure drop across the load as $P_l = P_1 - P_2$ and considering that for a symmetrical cylinder $A_p = A_1 = A_2$, Eq. 1 can be rewritten as

$$M_t \ddot{x} + B_t \dot{x} + K_s x = A_p P_l \tag{2}$$

Applying continuity equation to the fluid flow, the following equation is obtained:

$$Q_l = A_p \dot{x} + C_{tp} + \frac{V_t}{4\beta_e} \dot{P}_l \tag{3}$$

where $Q_l = (Q_1 + Q_2)/2$ is the load flow, C_{tp} the total leakage coefficient of piston, V_t the total volume under compression in both chambers and β_e the effective bulk modulus.

Considering that the return line pressure is usually much smaller than the other pressures involved ($P_0 \approx 0$) and assuming a closed center spool valve with matched and symmetrical orifices, the relationship between load pressure P_l and load flow Q_l can be described as follows

$$Q_l = C_d w \bar{x}_{sp} \sqrt{\frac{1}{\rho}\left(P_s - \mathrm{sgn}(\bar{x}_{sp}) P_l\right)} \tag{4}$$

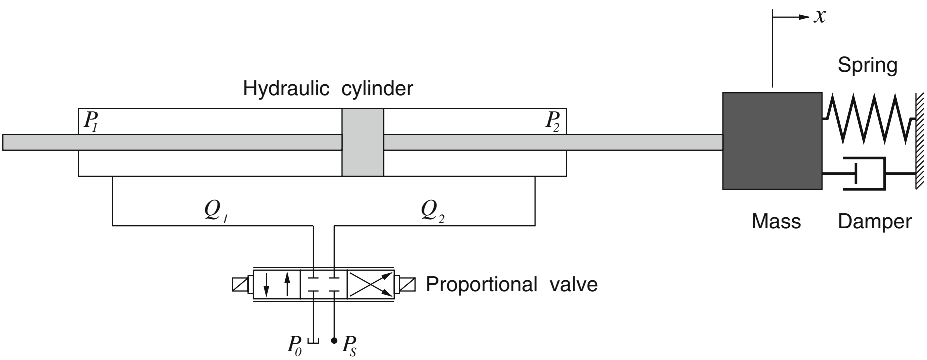


Fig. 1 Schematic diagram of the electro-hydraulic servo-system

where C_d is the discharge coefficient, w the valve orifice area gradient, $\bar{x}_\mathrm{sp}$ the effective spool displacement from neutral, ρ the hydraulic fluid density, P_s the supply pressure and $\mathrm{sgn}(\cdot)$ is defined by

$$\mathrm{sgn}(z) = \begin{cases} -1 & \text{if} \quad z < 0 \\ 0 & \text{if} \quad z = 0 \\ 1 & \text{if} \quad z > 0 \end{cases}$$

Assuming that the dynamics of the valve are fast enough to be neglected, the valve spool displacement can be considered as proportional to the control voltage (u). For closed center valves, or even in the case of the so-called critical valves, the spool presents some overlap. This overlap prevents from leakage losses but leads to a dead-zone nonlinearity within the control voltage, as shown in Fig. 2.

The adopted dead-zone model is a slightly modified version of that proposed by Zhang and Ge [21], which can be mathematically described by

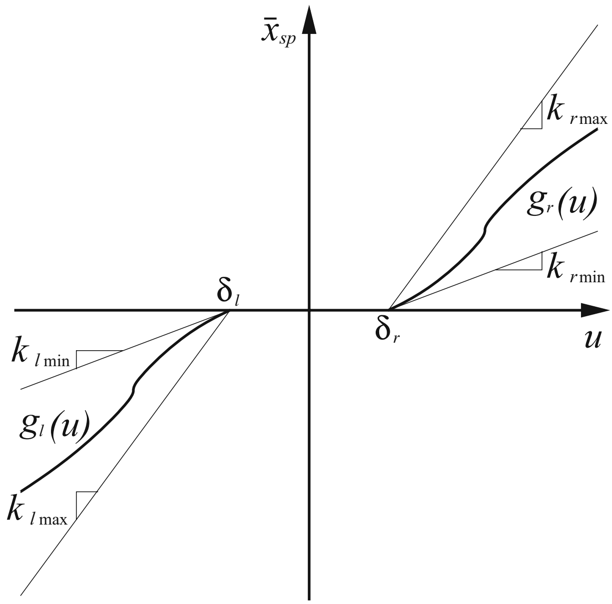
$$\bar{x}_\mathrm{sp} = \begin{cases} g_l(u) & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ g_r(u) & \text{if} \quad u \geq \delta_r \end{cases} \tag{5}$$

where g_l and g_r are functions of control voltage and the dead-band parameters δ_l and δ_r depends on the size of the overlap region.

In respect of the dead-zone model presented in Eq. 5, the following assumptions can be made:

Assumption 1 *The dead-zone output $\bar{x}_\mathrm{sp}$ is not available to be measured.*

Fig. 2 Dead-zone nonlinearity



Assumption 2 *The dead-band parameters δ_l and δ_r are unknown but bounded and with known signs, i.e., $\delta_{l\,\min} \leq \delta_l \leq \delta_{l\,\max} < 0$ and $0 < \delta_{r\,\min} \leq \delta_r \leq \delta_{r\,\max}$.*

Assumption 3 *The functions $g_l : (-\infty, \delta_l]$ and $g_r : [\delta_r, +\infty)$ are C^1 and with bounded positive-valued derivatives, i.e.,*

$$0 < k_{l\,\min} \leq g_l'(u) \leq k_{l\,\max}, \quad \forall u \in (-\infty, \delta_l],$$

$$0 < k_{r\,\min} \leq g_r'(u) \leq k_{r\,\max}, \quad \forall u \in [\delta_r, +\infty),$$

where $g_l'(u) = dg_l(z)/dz|_{z=u}$ and $g_r'(u) = dg_r(z)/dz|_{z=u}$.

Remark 1 Assumption 3 means that both g_l and g_r are Lipschitz functions.

From the mean value theorem and noting that $g_l(\delta_l) = g_r(\delta_r) = 0$, it follows that there exist $\xi_l : \mathbb{R} \to (-\infty, \delta_l)$ and $\xi_r : \mathbb{R} \to (\delta_r, +\infty)$ such that

$$g_l(u) = g_l'\big(\xi_l(u)\big)[u - \delta_l]$$

$$g_r(u) = g_r'\big(\xi_r(u)\big)[u - \delta_r]$$

In this way, Eq. 5 can be rewritten as follows:

$$\bar{x}_{\text{sp}} = \begin{cases} g_l'\big(\xi_l(u)\big)[u - \delta_l] & \text{if} \quad u \leq \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ g_r'\big(\xi_r(u)\big)[u - \delta_r] & \text{if} \quad u \geq \delta_r \end{cases} \tag{6}$$

or in a more appropriate form:

$$\bar{x}_{\text{sp}} = k_v(u)[u - d(u)] \tag{7}$$

where

$$k_v(u) = \begin{cases} g_l'\big(\xi_l(u)\big) & \text{if} \quad u \leq 0 \\ g_r'\big(\xi_r(u)\big) & \text{if} \quad u > 0 \end{cases} \tag{8}$$

and

$$d(u) = \begin{cases} \delta_l & \text{if} \quad u \leq \delta_l \\ u & \text{if} \quad \delta_l < u < \delta_r \\ \delta_r & \text{if} \quad u \geq \delta_r \end{cases} \tag{9}$$

Remark 2 Considering Assumption 2 and Eq. 9, it can be easily verified that $d(u)$ is bounded: $|d(u)| \leq \delta$, where $\delta = \max\{-\delta_{l\,\min}, \delta_{r\,\max}\}$.

Combining Eqs. 2, 3, 4 and 7 leads to a third-order differential equation that represents the dynamic behavior of the electro-hydraulic system:

$$\dddot{x} = -\mathbf{a}^{\mathrm{T}}\mathbf{x} + b(\mathbf{x}, u)u - b(\mathbf{x}, u)d(u) \tag{10}$$

where $\mathbf{x} = [x, \dot{x}, \ddot{x}]$ is the state vector with an associated coefficient vector $\mathbf{a} = [a_0, a_1, a_2]$ defined according to

$$a_0 = \frac{4\beta_{\mathrm{e}} C_{\mathrm{tp}} K_{\mathrm{s}}}{V_{\mathrm{t}} M_{\mathrm{t}}} \quad ; \quad a_1 = \frac{K_{\mathrm{s}}}{M_{\mathrm{t}}} + \frac{4\beta_{\mathrm{e}} A_p^2}{V_{\mathrm{t}} M_{\mathrm{t}}} + \frac{4\beta_{\mathrm{e}} C_{\mathrm{tp}} B_{\mathrm{t}}}{V_{\mathrm{t}} M_{\mathrm{t}}} \quad ; \quad a_2 = \frac{B_{\mathrm{t}}}{M_{\mathrm{t}}} + \frac{4\beta_{\mathrm{e}} C_{\mathrm{tp}}}{V_{\mathrm{t}}}$$

and

$$b(\mathbf{x},u)=\frac{4\beta_{\mathrm{e}}A_{\mathrm{p}}}{V_{\mathrm{t}}M_{\mathrm{t}}}C_{\mathrm{d}}wk_{v}\sqrt{\frac{1}{\rho}\left[P_{\mathrm{s}}-\operatorname{sgn}(u)\big(M_{\mathrm{t}}\ddot{x}+B_{\mathrm{t}}\dot{x}+K_{\mathrm{s}}x\big)/A_{\mathrm{p}}\right]}$$

In respect of the dynamic system presented in Eq. 10, the following assumptions will also be made:

Assumption 4 *The coefficients a_0, a_1 and a_2 are unknown but bounded: $|(\hat{\mathbf{a}}-\mathbf{a})^{\mathrm{T}}\mathbf{x}|\leq\alpha$, where $\hat{\mathbf{a}}$ is an estimate of $\mathbf{a}$.*

Assumption 5 *The input gain $b(\mathbf{x},u)$ is unknown but positive and bounded: $0<b_{\min}\leq b(\mathbf{x},u)\leq b_{\max}$.*

Based on the dynamic model presented in Eq. 10, an adaptive fuzzy sliding mode controller will be developed in the next section.

3 Adaptive Fuzzy Sliding Mode Controller

As shown by Bessa et al. [25], adaptive fuzzy algorithms can be properly embedded in smooth sliding mode controllers to improve the trajectory tracking of uncertain nonlinear systems. Adaptive fuzzy sliding mode controllers based on this strategy has already been successfully applied to the dynamic positioning of remotely operated underwater vehicles [26] and to the chaos control in a nonlinear pendulum [27].

Here, the proposed control problem is to ensure that, even in the presence of parametric uncertainties, unmodeled dynamics and an unknown dead-zone input, the state vector $\mathbf{x}$ will follow a desired trajectory $\mathbf{x}_{\mathrm{d}}=[x_{\mathrm{d}},\dot{x}_{\mathrm{d}},\ddot{x}_{\mathrm{d}}]$ in the state space.

Regarding the development of the control law, the following assumptions should also be made:

Assumption 6 *The state vector $\mathbf{x}$ is available.*

Assumption 7 *The desired trajectory $\mathbf{x}_{\mathrm{d}}$ is once differentiable in time. Furthermore, every element of vector $\mathbf{x}_{\mathrm{d}}$, as well as $\dddot{x}_{\mathrm{d}}$, is available and with known bounds.*

Let $\tilde{x}=x-x_{\mathrm{d}}$ be defined as the tracking error in the variable x, $\tilde{\mathbf{x}}=\mathbf{x}-\mathbf{x}_{\mathrm{d}}=[\tilde{x},\dot{\tilde{x}},\ddot{\tilde{x}}]$ as the tracking error vector and consider a sliding surface S defined in the state space by the equation $s(\tilde{\mathbf{x}})=0$, with the function $s:\mathbb{R}^3\rightarrow\mathbb{R}$ satisfying

$$s(\tilde{\mathbf{x}})=\ddot{\tilde{x}}+2\lambda\dot{\tilde{x}}+\lambda^2\tilde{x}\tag{11}$$

where λ is a strictly positive constant.

Now, the problem of controlling the system dynamics (Eq. 10) can be treated according to the sliding mode methodology, by defining a control law composed by an equivalent control $\hat{u} = \hat{b}^{-1}\big(\hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \dddot{x}_{\mathrm{d}} - 2\lambda\ddot{\tilde{x}} - \lambda^2\dot{\tilde{x}}\big)$, an estimate $\hat{d}(\hat{u})$ and a discontinuous term $-K\,\mathrm{sgn}(s)$:

$$u = \hat{b}^{-1}\big(\hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \dddot{x}_{\mathrm{d}} - 2\lambda\ddot{\tilde{x}} - \lambda^2\dot{\tilde{x}}\big) + \hat{d}(\hat{u}) - K\,\mathrm{sgn}(s) \tag{12}$$

where K is the control gain.

Based on Assumption 5 and considering that the estimate $\hat{b}$ could be chosen according to the geometric mean $\hat{b} = \sqrt{b_{\max} b_{\min}}$, the bounds of b may be expressed as $\gamma^{-1} \leq \hat{b}/b \leq \gamma$, where $\gamma = \sqrt{b_{\max}/b_{\min}}$.

Under this condition, the gain K should be chosen according to:

$$K \geq \gamma\hat{b}^{-1}(\eta + \alpha) + \delta + |\hat{d}(\hat{u})| + (\gamma - 1)|\hat{u}| \tag{13}$$

where η is a strictly positive constant related to the reaching time.

At this point, it should be highlighted that the control law (Eq. 12), together with Eq. 13, is sufficient to impose the sliding condition

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s|$$

and, consequently, the finite time convergence to the sliding surface S.

In order to obtain a good approximation to $d(u)$, the estimate $\hat{d}(\hat{u})$ will be computed directly by an adaptive fuzzy algorithm. The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in a linguistic manner as follows:

$$\textit{If } \hat{u} \textit{ is } \hat{U}_r \textit{ then } \hat{d}_r = \hat{D}_r\,, \quad r = 1, 2, \ldots, N$$

where $\hat{U}_r$ are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the N fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(\hat{u}) = \frac{\sum_{r=1}^{N} w_r \cdot \hat{d}_r}{\sum_{r=1}^{N} w_r}$$

or, similarly,

$$\hat{d}(\hat{u}) = \hat{\mathbf{D}}^{\mathrm{T}}\boldsymbol{\Psi}(\hat{u}) \tag{14}$$

where, $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]^{\mathrm{T}}$ is the vector containing the attributed values $\hat{D}_r$ to each rule r, $\boldsymbol{\Psi}(\hat{u}) = [\psi_1(\hat{u}), \psi_2(\hat{u}), \ldots, \psi_N(\hat{u})]$ is a vector with components $\psi_r(\hat{u}) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule.

To ensure the best possible estimate $\hat{d}(\hat{u})$, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = -\varphi s \boldsymbol{\Psi}(\hat{u}) \tag{15}$$

where φ is a strictly positive constant related to the adaptation rate.

It is important to emphasize that the chosen adaptation law, Eq. 15, must not only provide a good approximation to $d(u)$ but also assure the convergence of the state variables to the sliding surface $S(t)$, for the purpose of trajectory tracking. In this way, in order to evaluate the stability of the closed-loop system, let a positive-definite function V_1 be defined as

$$V_1(t) = \frac{1}{2}s^2 + \frac{b}{2\varphi}\boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Delta}$$

where $\boldsymbol{\Delta} = \hat{\mathbf{D}} - \hat{\mathbf{D}}^*$ and $\hat{\mathbf{D}}^*$ is the optimal parameter vector, associated to the optimal estimate $\hat{d}^*(\hat{u}) = d(u)$.

Thus, the time derivative of V_1 is

$$\begin{aligned}\dot{V}_1(t) &= s\dot{s} + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\dot{\boldsymbol{\Delta}} = \big(\dddot{\tilde{x}} + 2\lambda\ddot{\tilde{x}} + \lambda^2\dot{\tilde{x}}\big)s + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\dot{\boldsymbol{\Delta}}\\
&= \big(-\mathbf{a}^{\mathrm{T}}\mathbf{x} + bu - bd - \dddot{x}_{\mathrm{d}} + 2\lambda\ddot{\tilde{x}} + \lambda^2\dot{\tilde{x}}\big)s + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\dot{\boldsymbol{\Delta}}\\
&= \big[-\mathbf{a}^{\mathrm{T}}\mathbf{x} + b\hat{b}^{-1}\big(\hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \dddot{x}_{\mathrm{d}} - 2\lambda\ddot{\tilde{x}} - \lambda^2\dot{\tilde{x}}\big) + b\hat{d} - bK\,\mathrm{sgn}(s)\\
&\qquad - bd - \dddot{x}_{\mathrm{d}} + 2\lambda\ddot{\tilde{x}} + \lambda^2\dot{\tilde{x}}\big]s + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\dot{\boldsymbol{\Delta}}\end{aligned}$$

Defining a minimum approximation error as $\varepsilon = \hat{d}^* - d$, recalling that $\hat{u} = \hat{b}^{-1}(\hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \dddot{x}_{\mathrm{d}} - 2\lambda\ddot{\tilde{x}} - \lambda^2\dot{\tilde{x}})$ and noting that $\dot{\boldsymbol{\Delta}} = \dot{\hat{\mathbf{D}}}$ and $\mathbf{a}^{\mathrm{T}}\mathbf{x} = \hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} - (\hat{\mathbf{a}} - \mathbf{a})^{\mathrm{T}}\mathbf{x}$, $\dot{V}_1$ becomes:

$$\begin{aligned}\dot{V}_1(t) &= -\big[bK\,\mathrm{sgn}(s) - (\hat{\mathbf{a}} - \mathbf{a})^{\mathrm{T}}\mathbf{x} + \hat{b}\hat{u} - b\hat{u} - b\varepsilon - b(\hat{d} - \hat{d}^*)\big]s + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}}\\
&= -\big[bK\,\mathrm{sgn}(s) - (\hat{\mathbf{a}} - \mathbf{a})^{\mathrm{T}}\mathbf{x} + \hat{b}\hat{u} - b\hat{u} - b\varepsilon - b\,\boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Psi}(\hat{u})\big]s + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}}\\
&= -\big[bK\,\mathrm{sgn}(s) - (\hat{\mathbf{a}} - \mathbf{a})^{\mathrm{T}}\mathbf{x} + \hat{b}\hat{u} - b\hat{u} - b\varepsilon\big]s + b\varphi^{-1}\boldsymbol{\Delta}^{\mathrm{T}}\big[\dot{\hat{\mathbf{D}}} + \varphi s\boldsymbol{\Psi}(\hat{u})\big]\end{aligned}$$

Thus, by applying the adaptation law (Eq. 15) to $\dot{\hat{\mathbf{D}}}$:

$$\dot{V}_1(t) = -[bK\,\mathrm{sgn}(s) - (\hat{\mathbf{a}} - \mathbf{a})^{\mathrm{T}}\mathbf{x} + \hat{b}\hat{u} - b\hat{u} - b\,\varepsilon]s$$

Furthermore, considering Assumptions 2–5, defining K according to Eq. 13 and verifying that $|\varepsilon| = |\hat{d}^* - d| \leq |\hat{d} - d| \leq |\hat{d}| + \delta$, it follows that

$$\dot{V}_1(t) \leq -\eta|s| \tag{16}$$

which implies $V_1(t) \leq V_1(0)$ and that s and $\boldsymbol{\Delta}$ are bounded. The definition of s, Eq. 11, implies that $\tilde{\mathbf{x}}$ is bounded. On this basis, from the definition of $\dot{s}$, $\dot{s}(\tilde{\mathbf{x}}) = \dddot{\tilde{x}} + 2\lambda\ddot{\tilde{x}} + \lambda^2\dot{\tilde{x}}$, and Assumption 7 it can be verified that $\dot{s}$ is also bounded.

Integrating both sides of Eq. 16 shows that

$$\lim_{t\to\infty}\int_0^t \eta|s|\,d\tau \leq \lim_{t\to\infty}[V_1(0) - V_1(t)] \leq V_1(0) < \infty$$

Since the absolute value function is uniformly continuous, it follows from Barbalat's lemma [28] that $s \to 0$ as $t \to \infty$, which ensures the convergence of the tracking error vector to the sliding surface S.

In spite of the demonstrated properties of the controller, the presence of a discontinuous term in the control law leads to the well known chattering phenomenon. In order to overcome the undesirable chattering effects, a thin boundary layer, S_ϕ, in the neighborhood of the switching surface can be adopted [29]:

$$S_\phi = \big\{\tilde{\mathbf{x}} \in \mathbb{R}^3 \;\big|\; |s(\tilde{\mathbf{x}})| \leq \phi\big\}$$

where ϕ is a strictly positive constant that represents the boundary layer thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside S_ϕ. It should be noted that this smooth approximation must behave exactly like the sign function outside the boundary layer. There are several options to smooth out the ideal relay but the most common choice is the saturation function:

$$\mathrm{sat}(s/\phi) = \begin{cases} \mathrm{sgn}(s) & \text{if} \quad |s/\phi| \geq 1 \\ s/\phi & \text{if} \quad |s/\phi| < 1 \end{cases}$$

In this way, to avoid chattering, a smooth version of Eq. 12 is defined:

$$u = \hat{b}^{-1}\big(\hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \dddot{x}_{\mathrm{d}} - 2\lambda\ddot{\tilde{x}} - \lambda^2\dot{\tilde{x}}\big) + \hat{d}(\hat{u}) - K\,\mathrm{sat}(s/\phi) \tag{17}$$

In order to establish the attractiveness and invariant properties of the defined boundary layer, let a new Lyapunov function candidate V_2 be defined as

$$V_2(t) = \frac{1}{2}s_\phi^2$$

where s_ϕ is a measure of the distance of the current state to the boundary layer, and can be computed as follows

$$s_\phi = s - \phi \ \mathrm{sat}(s/\phi) \tag{18}$$

Noting that $s_\phi = 0$ inside the boundary layer and $\dot{s}_\phi = \dot{s}$, one has $\dot{V}_2(t) = 0$ inside S_ϕ, and outside

$$\dot{V}_2(t) = s_\phi\dot{s}_\phi = s_\phi\dot{s} = \big(\dddot{\tilde{x}} + 2\lambda\ddot{\tilde{x}} + \lambda^2\dot{\tilde{x}}\big)s_\phi = \big(-\mathbf{a}^{\mathrm{T}}\mathbf{x} + bu - bd - \dddot{x}_{\mathrm{d}} + 2\lambda\ddot{\tilde{x}} + \lambda^2\dot{\tilde{x}}\big)s_\phi$$

It can be easily verified that outside the boundary layer the control law (Eq. 17) takes the following form:

$$u = \hat{b}^{-1}\big(\hat{\mathbf{a}}^{\mathrm{T}}\mathbf{x} + \dddot{x}_{\mathrm{d}} - 2\lambda\ddot{\tilde{x}} - \lambda^2\dot{\tilde{x}}\big) + \hat{d}(\hat{u}) - K\,\mathrm{sgn}(s_\phi)$$

Thus, the time derivative $\dot{V}_2$ can be written as

$$\dot{V}_2(t) = -\big[bK\,\mathrm{sgn}(s_\phi) - (\hat{\mathbf{a}} - \mathbf{a})^{\mathrm{T}}\mathbf{x} + \hat{b}\hat{u} - b\hat{u} - b\hat{d} + bd\big]s_\phi$$

Therefore, by considering Assumptions 2–5 and defining K according to Eq. 13, $\dot{V}_2(t)$ becomes:

$$\dot{V}_2(t) \leq -\eta|s_\phi| \tag{19}$$

which implies $V_2(t) \leq V_2(0)$ and that s_ϕ is bounded. The definitions of s and s_ϕ, respectively Eqs. 11 and 18, implies that $\tilde{\mathbf{x}}$ is bounded. From the definition of $\dot{s}$ and Assumption 7 it can be verified that $\dot{s}$ is also bounded.

The finite-time convergence of the states to the boundary layer can be shown by integrating both sides of Eq. 19 over the interval $0 \leq t \leq t_{\mathrm{reach}}$, where t_{reach} is the time required to hit S_ϕ. In this way, noting that $|s_\phi(t_{\mathrm{reach}})| = 0$, one has:

$$t_{\mathrm{reach}} \leq \frac{|s_\phi(0)|}{\eta} \tag{20}$$

which guarantees the convergence of the tracking error vector to the boundary layer in a time interval smaller than $|s_\phi(0)|/\eta$.

Nevertheless, it should be emphasized that the substitution of the discontinuous term by a smooth approximation inside the boundary layer turns the perfect tracking into a tracking with guaranteed precision problem, which actually means that a steady-state error will always remain. However, it can be verified that, once inside the boundary layer, the tracking error vector will exponentially converge to a closed region Φ.

Considering that $|s(\tilde{\mathbf{x}})| \leq \phi$ may be rewritten as $-\phi \leq s(\tilde{\mathbf{x}}) \leq \phi$ and from the definition of $s(\tilde{\mathbf{x}})$, Eq. 11, one has:

$$-\phi \leq \ddot{\tilde{x}} + 2\lambda\dot{\tilde{x}} + \lambda^2\tilde{x} \leq \phi \tag{21}$$

Multiplying Eq. 21 by $e^{\lambda t}$ and integrating between 0 and t:

$$\begin{aligned}
-\phi e^{\lambda t} &\leq \left(\ddot{\tilde{x}} + 2\lambda\dot{\tilde{x}} + \lambda^2\tilde{x}\right)e^{\lambda t} \leq \phi e^{\lambda t} \\
-\phi e^{\lambda t} &\leq \frac{d^2}{dt^2}(\tilde{x}e^{\lambda t}) \leq \phi e^{\lambda t} \\
-\phi \int_0^t e^{\lambda\tau} d\tau &\leq \int_0^t \frac{d^2}{d\tau^2}(\tilde{x}e^{\lambda\tau})d\tau \leq \phi \int_0^t e^{\lambda\tau} d\tau \\
-\frac{\phi}{\lambda}e^{\lambda t} + \frac{\phi}{\lambda} &\leq \frac{d}{dt}(\tilde{x}e^{\lambda t}) - \left.\frac{d}{dt}(\tilde{x}e^{\lambda t})\right|_{t=0} \leq \frac{\phi}{\lambda}e^{\lambda t} - \frac{\phi}{\lambda}
\end{aligned}$$

or conveniently rewritten as

$$-\frac{\phi}{\lambda}e^{\lambda t} - \left(\left|\frac{d}{dt}(\tilde{x}e^{\lambda t})\right|_{t=0} + \frac{\phi}{\lambda}\right) \leq \frac{d}{dt}(\tilde{x}e^{\lambda t}) \leq \frac{\phi}{\lambda}e^{\lambda t} + \left(\left|\frac{d}{dt}(\tilde{x}e^{\lambda t})\right|_{t=0} + \frac{\phi}{\lambda}\right) \tag{22}$$

Now, integrating Eq. 22 between 0 and t:

$$\begin{aligned}
&-\frac{\phi}{\lambda^2}e^{\lambda t} - \left(\left|\frac{d}{dt}(\tilde{x}e^{\lambda t})\right|_{t=0} + \frac{\phi}{\lambda}\right)t \\
&\quad - \left(|\tilde{x}(0)| + \frac{\phi}{\lambda^2}\right) \leq \tilde{x}e^{\lambda t} \leq \frac{\phi}{\lambda^2}e^{\lambda t} \\
&\quad + \left(\left|\frac{d}{dt}(\tilde{x}e^{\lambda t})\right|_{t=0} + \frac{\phi}{\lambda}\right)t + \left(|\tilde{x}(0)| + \frac{\phi}{\lambda^2}\right)
\end{aligned} \tag{23}$$

Furthermore, dividing Eq. 23 by $e^{\lambda t}$, it can be easily verified that for $t \to \infty$:

$$-\frac{\phi}{\lambda^2} \leq \tilde{x} \leq \frac{\phi}{\lambda^2} \tag{24}$$

By imposing the bounds (24) to (22), noting that $d(\tilde{x}e^{\lambda t})/dt = \dot{\tilde{x}}e^{\lambda t} + \tilde{x}\lambda e^{\lambda t}$ and dividing again by $e^{\lambda t}$, it follows that, for $t \to \infty$,

$$-2\frac{\phi}{\lambda} \leq \dot{\tilde{x}} \leq 2\frac{\phi}{\lambda} \tag{25}$$

Finally, applying Eqs. 24 and 25 to Eq. 21, one has

$$-6\phi \leq \ddot{\tilde{x}} \leq 6\phi \tag{26}$$

In this way, the tracking error will be confined within the limits $|\tilde{x}| \leq \phi/\lambda^2$, $|\dot{\tilde{x}}| \leq 2\phi/\lambda$ and $|\ddot{\tilde{x}}| \leq 6\phi$. However, these bounds define a box that is not completely inside the boundary layer. Considering the demonstrated attractiveness and invariant properties of S_ϕ, the region of convergence can be stated as the intersection of the boundary layer and the box defined by the preceding bounds. Therefore, it follows

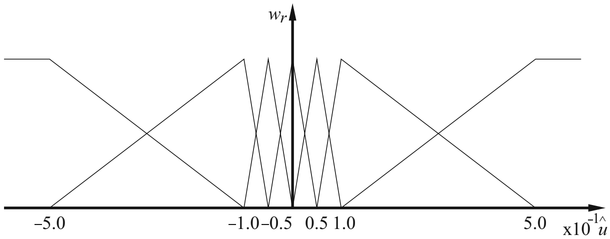


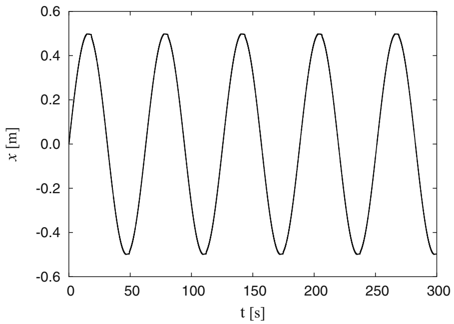
Fig. 3 Adopted fuzzy membership functions

that the tracking error vector will exponentially converge to a closed region $\Phi = \{\tilde{\mathbf{x}} \in \mathbb{R}^3 \mid |s(\tilde{\mathbf{x}})| \leq \phi$ and $|\tilde{x}| \leq \phi/\lambda^2$ and $|\dot{\tilde{x}}| \leq 2\phi/\lambda$ and $|\ddot{\tilde{x}}| \leq 6\phi\,\}$. It should be highlighted that the convergence region Φ is in perfect accordance with the bounds proposed by Bessa [30] for n^{th}-order nonlinear systems subject to smooth sliding mode controllers.

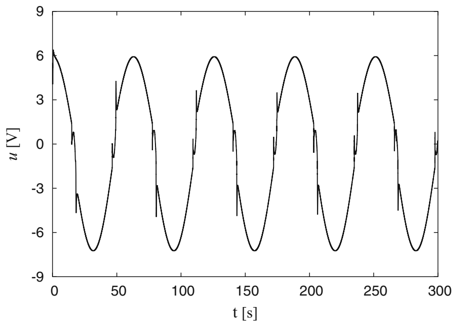
In the following section some numerical simulations are presented in order to evaluate the performance of the adaptive fuzzy sliding mode controller.

4 Simulation Results

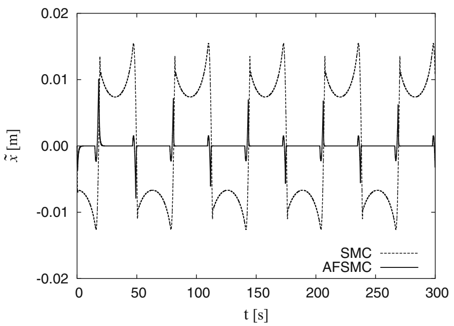
The simulation studies were performed with a numerical implementation in C, with sampling rates of 400 Hz for control system and 800 Hz for dynamic model. The



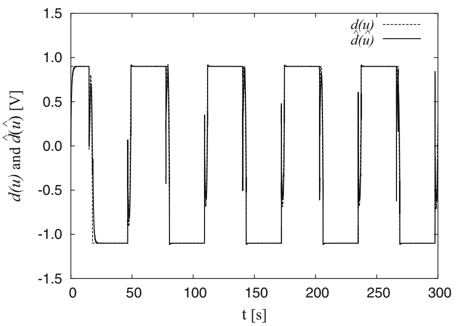
(a) Tracking performance.



(b) Control voltage.



(c) Tracking error.



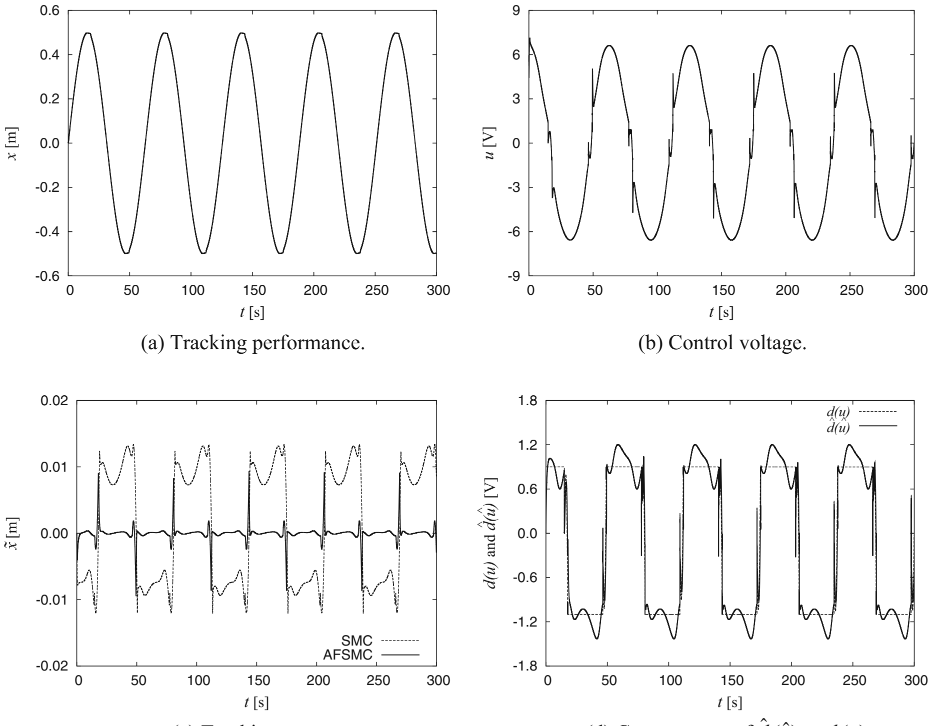
(d) Convergence of $\hat{d}\,(\hat{u})$ to $d\,(u)$.

Fig. 4 a–d Tracking performance with unknown dead-band and well known model parameters differential equations of the dynamic model were numerically solved with the fourth order Runge–Kutta method. In order to evaluate the control system performance, two numerical simulations were carried out.

In the first case, linear functions were taken into account for both $g_l(u) = k_l(u - \delta_l)$ and $g_r(u) = k_r(u - \delta_r)$. In this way, the adopted parameters for the electro-hydraulic system were $P_s = 7$ MPa, $\rho = 850$ kg/m^3, $C_d = 0.6$, $w = 2.5 \times 10^{-2}$ m, $A_p = 3 \times 10^{-4}$ m^2, $C_{tp} = 2 \times 10^{-12}$ m^3/(s Pa), $\beta_e = 700$ MPa, $V_t = 6 \times 10^{-5}$ m^3, $M_t = 250$ kg, $B_t = 100$ Ns/m, $K_s = 75$ N/m, $k_l = 1.8 \times 10^{-6}$ m/V, $k_r = 2.2 \times 10^{-6}$ m/V, $\delta_l = -1.1$ V and $\delta_r = 0.9$ V.

Assuming that the model parameters were perfectly known and considering the dead-zone width unknown, the following values were chosen for controller parameters $\lambda = 8$, $\varphi = 4$, $\gamma = 1.2$, $\delta = 1.1$, $\phi = 1$, $\eta = 0.1$ and $\alpha = 0$. For the fuzzy inference system, triangular and trapezoidal membership functions were adopted for $\hat{U}_r$, with central values defined as $C = \{-5.0\,;\ -1.0\,;\ -0.5\,;\ 0.0\,;\ 0.5\,;\ 1.0\,;\ 5.0\} \times 10^{-1}$ (see Fig. 3). It is also important to emphasize, that the vector of adjustable parameters was initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law presented in Eq. 15. Figure 4 shows the obtained results for the tracking of $x_d = 0.5\sin(0.1t)$ m.

As observed in Fig. 4, the adaptive fuzzy sliding mode controller (AFSMC) is able to provide trajectory tracking with small associated error and no chattering at all. It



(a) Tracking performance. (b) Control voltage.

(c) Tracking error. (d) Convergence of $\hat{d}\,(\hat{u})$ to $d\,(u)$.

Fig. 5 **a–d** Tracking performance with unknown dead-zone and uncertain model parameters can be also verified that the proposed control law leads to a smaller tracking error when compared with the conventional sliding mode controller (SMC), Fig. 4c. The improved performance of AFSMC over SMC is due to its ability to compensate for dead-zone effects, Fig. 4d. The AFSMC can be easily converted to the classical SMC by setting the adaptation rate to zero, $\varphi = 0$.

In the second simulation study it was assumed that the model parameters were not exactly known and nonlinear functions were assumed for $g_l(u) = k_l(u + 0.2\sin u - \delta_l)$ and $g_r(u) = k_r(u - 0.2\cos u - \delta_r)$, with $k_l = k_r = 2 \times 10^{-6}$ m/V. On this basis, considering a maximal uncertainty of $\pm 10\%$ over the value of k_v and variations of $\pm 20\%$ in the supply pressure, $P_\mathrm{s} = 7[1 + 0.2\sin(x)]$ MPa, the estimates $\hat{k}_v = 2 \times 10^{-6}$ m/V and $\hat{P}_\mathrm{s} = 7$ MPa were chosen for the computation of $\hat{b}$ in the control law. The other model and controller parameters, as well as the desired trajectory, were chosen as before. The obtained results are presented in Fig. 5.

Despite the unknown dead-zone input and uncertainties with respect to model parameters, the AFSMC allows the electro-hydraulic actuator to track the desired trajectory with a small tracking error, (see Fig. 5). As before, the undesirable chattering effect is not observed, Fig. 5b. Through the comparative analysis shown in Fig. 5c, the improved performance of the AFSMC over the uncompensated counterpart can be also clearly ascertained.

5 Concluding Remarks

The present work addressed the problem of controlling electro-hydraulic systems subject to an unknown dead-zone. An adaptive fuzzy sliding mode controller was implemented to deal with the trajectory tracking problem. The boundedness and convergence properties of the closed-loop systems was proven using Lyapunov stability theory and Barbalat's lemma. The control system performance was also confirmed by means of numerical simulations. The adaptive algorithm could automatically recognize the dead-zone nonlinearity and previously compensate for its undesirable effects.

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