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Adaptive fuzzy sliding mode control of a chaotic pendulum with noisy signals

Wallace M. Bessa[1,*], **Aline S. de Paula**[2,**], **and Marcelo A. Savi**[3,***]

[1] Universidade Federal do Rio Grande do Norte, Departamento de Engenharia Mecânica, Campus Universitário Lagoa Nova, CEP 59072-970, Natal, RN, Brazil
[2] Universidade de Brasília, Departamento de Engenharia Mecânica, CEP 70910-900, Brasília, DF, Brazil
[3] Universidade Federal do Rio de Janeiro, Departamento de Engenharia Mecânica, P.O. Box 68.503, CEP 21941-972, Rio de Janeiro, RJ, Brazil

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Chaotic response is related to a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaos has sensitive dependence to initial conditions, which implies that the system evolution may be altered by small perturbations. Chaos control is based on the richness of chaotic behavior and may be understood as the use of tiny perturbations for the stabilization of an UPO enclosed in a chaotic attractor. It makes this kind of behavior to be desirable in a variety of applications, since one of these UPOs can provide better performance than others in a particular situation. In this work, an adaptive fuzzy sliding mode controller is combined with the close return method for the stabilization of UPOs in a chaotic pendulum. The adaptive fuzzy inference system is embedded in a smooth sliding mode controller to cope with both structured and unstructured uncertainties. Since noise contamination is unavoidable in experimental data acquisition, this work also investigates the effect of noisy signals on the used control law, verifying their influence on the system stabilization and on the required control action. Numerical results are presented in order to illustrate the ability of the proposed control scheme to track UPOs even in the presence of modeling inaccuracies and noisy input signals.

1 Introduction

Chaos control may be understood as the use of tiny perturbations for the stabilization of unstable periodic orbits (UPOs) embedded in a chaotic attractor. The idea that chaotic behavior may be controlled by small perturbations of physical parameters allows this kind of behavior to be desirable in different applications.

The pioneer work of Ott et al. [25] established the fundamental ideas of chaos control. The OGY method, which received this name as a tribute to the authors Ott-Grebogi-Yorke, is a discrete technique that considers small perturbations applied in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section. Afterward, the delayed feedback control [27] was proposed as a continuous method, establishing that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and the delayed state of the system. Since the beginning of chaos control studies in the 1990's, many alternative methods were proposed in order to overcome some limitations of the original techniques. Pyragas [28] presented a review about improvements and applications of time-delayed feedback control. Some advances in OGY method are also suggested in [9, 10, 13, 17, 24, 35]. Savi et al. [30] discussed some of these alternatives, and De Paula and Savi [11] presented a comparative analysis of chaos control methods.

Due to its robustness against modeling inaccuracies and external disturbance, sliding mode control has been successfully applied to chaotic systems [16, 19, 37, 38]. But a well known handicap of conventional sliding mode controllers is the chattering effect. To overcome the undesired effects of the control chattering, Slotine [34] proposed the adoption of a thin boundary layer neighboring the switching surface, by replacing the sign function by a saturation function. This substitution can minimize or, when desired, even completely eliminate chattering, but turns perfect tracking into a tracking with guaranteed precision problem, which in fact means that a steady-state error will always remain.

* Corresponding author E-mail: wmbessa@ufrnet.br, Phone: +55 84 3215 3740, Fax: +55 84 3215 3768
** E-mail: alinedepaula@unb.br
*** E-mail: savi@mecanica.ufrj.br

In order to enhance the tracking performance, Bessa et al. [3] used an adaptive fuzzy inference system to approximate the unknown system dynamics within the boundary layer. The resulting adaptive fuzzy sliding mode scheme is applied to the chaos control of a nonlinear pendulum. However, the adoption of the state variables in the premise of the fuzzy rules is a drawback of this approach. As for instance, for higher-order systems the number of fuzzy sets and fuzzy rules becomes incredibly large, which compromises the applicability of this technique. To reduce the number of fuzzy sets and rules and consequently simplify the design process, Bessa and Barrêto [2] proposed the adoption of only one variable, the switching variable s, instead of the state variables in the premise of the fuzzy rules.

In this work, the adaptive fuzzy sliding mode controller presented in [2] is joined with the close return method [1] for the stabilization of UPOs in a chaotic pendulum. Numerical simulations are carried out to evaluate the performance of the adopted control scheme. Unstructured uncertainties related to unmodeled dynamics and structured uncertainties associated with parametric variations are both considered in the robustness analysis. Moreover, the analysis from noisy time series is conducted showing the effectiveness of the controller to stabilize unstable orbits.

2 Chaotic pendulum

The chaotic pendulum investigated here is based on an experimental set up, previously analyzed by Franca and Savi [15] and Pereira-Pinto et al. [26]. De Paula et al. [12] presented a mathematical model to describe the dynamical behavior of the pendulum and the corresponding experimentally obtained parameters.

The schematic picture of the considered nonlinear pendulum is shown in Fig. 1. Basically, the pendulum consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). This assembly is driven by a string-spring device (6) that is attached to an electric motor (7) and also provides torsional stiffness to the system. A magnetic device (3) provides an adjustable dissipation of energy. An actuator (5) provides the necessary perturbations to stabilize this system by properly changing the string length.

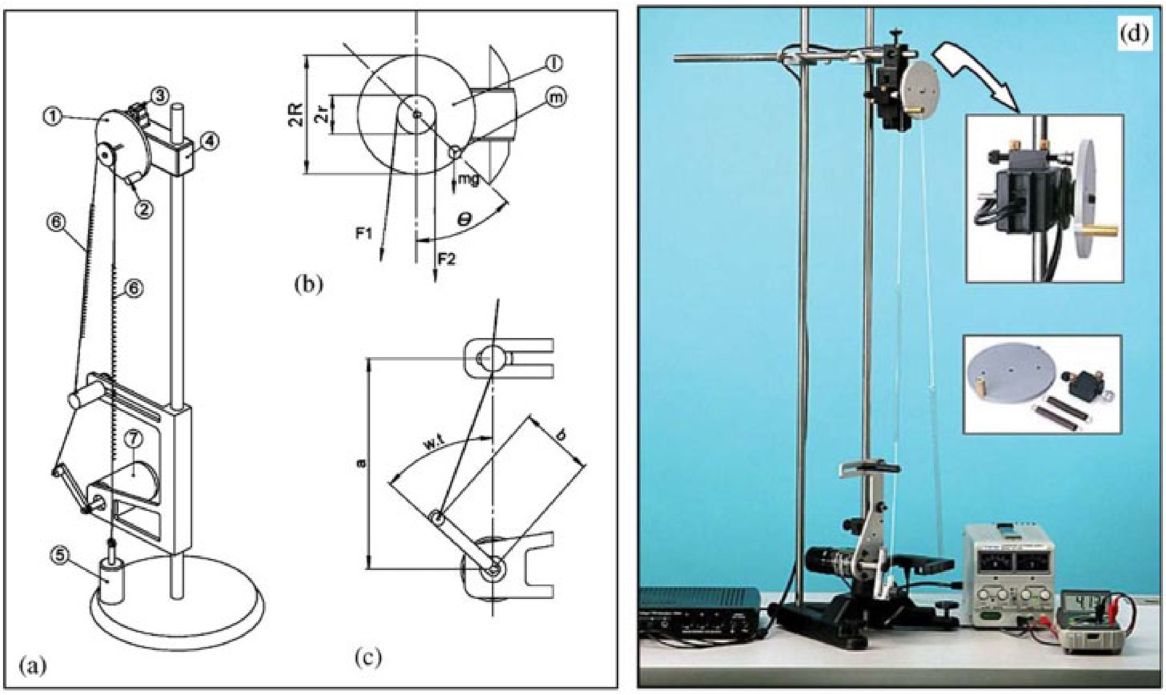


Fig. 1 (a) Nonlinear pendulum – (1) metallic disc; (2) lumped mass; (3) magnetic damping device; (4) rotary motion sensor (PASCO CI-6538); (5) anchor mass; (6) string-spring device; (7) electric motor (PASCO ME-8750). (b) Parameters and forces on metallic disc. (c) Parameters from driving device. (d) Experimental apparatus.

In order to obtain the equations of motion of the experimental nonlinear pendulum it is assumed that system dissipation may be expressed by a combination of a linear viscous dissipation together with dry friction. Therefore, denoting the angular position as θ, the following equation is obtained [12]:

$$I\ddot{\theta} + \zeta\dot{\theta} + \mu\,\mathrm{sgn}(\dot{\theta}) + 2kr^2\theta + mgR\sin(\theta) = kr\big[\sqrt{a^2 + b^2 - 2ab\cos(\omega t)} - (a - b) - \Delta l\big], \tag{1}$$

where ω is the forcing frequency related to the motor rotation, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation crank of the motor, R is the radius of the metallic disc, and r is the radius of the driving pulley, m is the lumped mass, ζ represents the linear viscous damping coefficient, while μ is the dry friction coefficient; g is the gravity acceleration, I is the inertia of the disk-lumped mass, k is the string stiffness, Δl is the length variation in the spring provided by the linear actuator (5) and sgn($\dot{\theta}$) is defined as

$$\mathrm{sgn}(\theta) = \begin{cases} -1 & \text{if} \quad \dot{\theta} < 0, \\ 0 & \text{if} \quad \dot{\theta} = 0, \\ 1 & \text{if} \quad \dot{\theta} > 0. \end{cases} \tag{2}$$

De Paula et al. [12] showed that this mathematical model presents results that are in close agreement with experimental data.

Here, for control purposes, Eq. (1) is conveniently rewritten as

$$\ddot{\theta} = f(\theta, \dot{\theta}, t) + hu + d \tag{3}$$

with $f = \left[kr\sqrt{a^2 + b^2 - 2ab\cos(\omega t)} - kr(a-b) - \zeta\dot{\theta} - \mu\,\mathrm{sgn}(\dot{\theta}) - 2kr^2\theta - mgR\sin(\theta)\right]/I$, $h = kr/I$, $u = -\Delta l$ and d represents unmodeled dynamics and external disturbances that can arise.

Concerning the dynamic system presented in Eq. (3), the following assumptions are made:

Assumption 2.1 The function f is unknown but bounded, i.e. $|\hat{f}(\theta, \dot{\theta}, t) - f(\theta, \dot{\theta}, t)| \leq \mathcal{F}$, where $\hat{f}$ is an estimate of f.

Assumption 2.2 The input gain h is unknown but positive and bounded, i.e. $0 < h_{\min} \leq h \leq h_{\max}$.

Assumption 2.3 The disturbance d is unknown but bounded, i.e. $|d| \leq \mathcal{D}$.

3 Controlling the chaotic pendulum

As demonstrated by Bessa and Barrêto [2], adaptive fuzzy algorithms can be properly embedded in smooth sliding mode controllers to compensate for modeling inaccuracies, in order to improve the trajectory tracking of uncertain nonlinear systems. It has also been shown that adaptive fuzzy sliding mode controllers are suitable for a variety of applications ranging from remotely operated underwater vehicles [4, 5] to electro-hydraulic servo-systems [6].

On this basis, the proposed control problem is to ensure that, even in the presence of modeling inaccuracies, external disturbances and noisy measurements, the state vector $\boldsymbol{\theta} = [\theta, \dot{\theta}]$ follows the desired unstable periodic orbit $\boldsymbol{\theta}_d = [\theta_d, \dot{\theta}_d]$. The UPOs are identified using the close return method [1]. In this regard, considering a time series represented by vectors $\{u_i\}_{i=1}^N$, the identification of a period-P UPO is based on a search for pairs of points in the time series that satisfy the condition $|u_i - u_{i+P}|_{i=1}^{N-P} \leq \delta$, where δ is the tolerance value for distinguishing return points. After this analysis, all points that belong to a period-P cycle are grouped together. During the search, the vicinity of an UPO may be visited many times, and it is necessary to distinguish each orbit, remove any cycle permutation and to average them in order to improve estimations as shown by Otani and Jones [24].

Now, according to the control strategy described in [2] and considering the switching variable as $s = \dot{\tilde{\theta}} + \lambda\tilde{\theta}$, with $\tilde{\theta} = \theta - \theta_d$ as the tracking error, the adaptive fuzzy sliding mode controller for the chaotic pendulum can be defined as follows

$$u = \hat{h}^{-1}(-\hat{f} + \ddot{\theta}_d - \lambda\dot{\tilde{\theta}}) + \hat{d}(s) - K\,\mathrm{sat}(s/\phi), \tag{4}$$

where $\hat{d}$ is an estimate of d, $\hat{h} = \sqrt{h_{\max}h_{\min}}$ is an estimate of h, K is a positive gain, ϕ is a strictly positive constant that represents the boundary layer thickness, and sat(s/ϕ) is defined as

$$\mathrm{sat}(s/\phi) = \begin{cases} \mathrm{sgn}(s) & \text{if} \quad |s/\phi| \geq 1, \\ s/\phi & \text{if} \quad |s/\phi| < 1. \end{cases}$$

Based on Assumptions 2.1–2.3 and considering that $\mathcal{H}^{-1} \leq \hat{h}/h \leq \mathcal{H}$, where $\mathcal{H} = \sqrt{h_{\max}/h_{\min}}$, the gain K should be chosen according to [2]:

$$K \geq \mathcal{H}\hat{h}^{-1}(\eta + \mathcal{F} + \mathcal{D} + |\hat{d}|) + (\mathcal{H} - 1)|\hat{u}|, \tag{5}$$

where η is a strictly positive constant related to the reaching time and $\hat{u} = \hat{h}^{-1}(-\hat{f} + \ddot{\theta}_d - \lambda\dot{\tilde{\theta}})$ is the equivalent control.

In order to obtain a good approximation to the disturbance d, the estimate $\hat{d}$ is computed directly by an adaptive fuzzy algorithm. According to [20], fuzzy systems can be considered as universal approximator, and hence, they can approximate any function on a compact set to an arbitrary accuracy.

The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang) [18], whose rules can be stated in a linguistic manner as follows:

$$\textit{If } s \textit{ is } S_r \textit{ then} \quad \hat{d}_r = \hat{D}_r; \quad r = 1, 2, \ldots, N,$$

where S_r are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the N fuzzy rules.

At this point, it should be highlighted that the adoption of the switching variable s in the premise of the rules, instead of the state variables as in [3], leads to a smaller number of fuzzy sets and rules, which simplifies the design process. Considering that external disturbances are independent of the state variables, the choice of a combined tracking error measure s also seems to be more appropriate in this case.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(s) = \hat{\mathbf{D}}^{\mathrm{T}} \mathbf{\Psi}(s), \tag{6}$$

where $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]$ is the vector containing the attributed values $\hat{D}_r$ to each rule r,

$$\mathbf{\Psi}(s) = \left[\psi_1(s), \psi_2(s), \ \ldots, \psi_N(s)\right]$$

is a vector with components $\psi_r(s) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule.

To ensure the best possible estimate $\hat{d}(s)$ to the disturbance d, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = \vartheta s \mathbf{\Psi}(s), \tag{7}$$

where ϑ is a strictly positive constant related to the adaptation rate.

A detailed discussion on the boundedness of all closed-loop signals and the convergence properties of the adaptive fuzzy sliding mode control of n^{th}-order uncertain nonlinear systems is presented in [2].

4 Tracking unstable periodic orbits

The controller capability is now investigated by considering numerical simulations. The fourth order Runge-Kutta method is employed and sampling rates of 107 Hz for control system and 214 Hz for dynamical model are assumed. The model parameters are chosen according to [12]: $I = 1.738 \times 10^{-4}$ kg m^2; $m = 1.47 \times 10^{-2}$ kg; $k = 2.47$ N/m; $\zeta = 2.368 \times 10^{-5}$ kg m^2/s; $\mu = 1.272 \times 10^{-4}$ N m; $a = 1.6e \times 10^{-1}$ m; $b = 6.0 \times 10^{-2}$ m; $r = 2.4 \times 10^{-2}$ m; $R = 4.75 \times 10^{-2}$ m and $\omega = 5.61$ rad/s.

Concerning the fuzzy system, triangular and trapezoidal membership functions are adopted for S_r, with the central values defined respectively as $C = \{-1.0\,;\,-0.2\,;\,-0.1\,;\,0.0\,;\,0.1\,;\,0.2\,;\,1.0\}$ (see Fig. 2). It is also important to emphasize that the vector of adjustable parameters is initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law, Eq. (7).

In order to demonstrate that the adopted control scheme can deal with both structured (parametric) and unstructured uncertainties (unmodeled dynamics), an uncertainty of $\pm 20\%$ over the value of the viscous damping coefficient, ζ, is

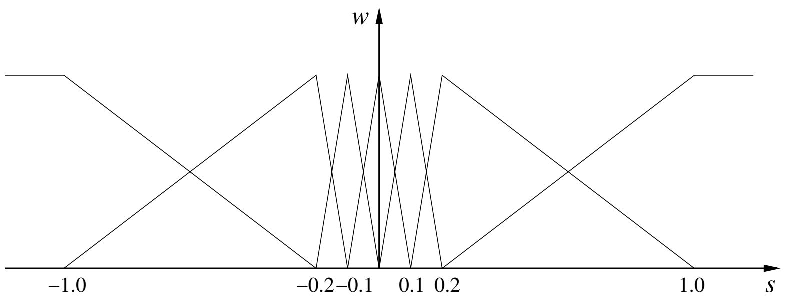
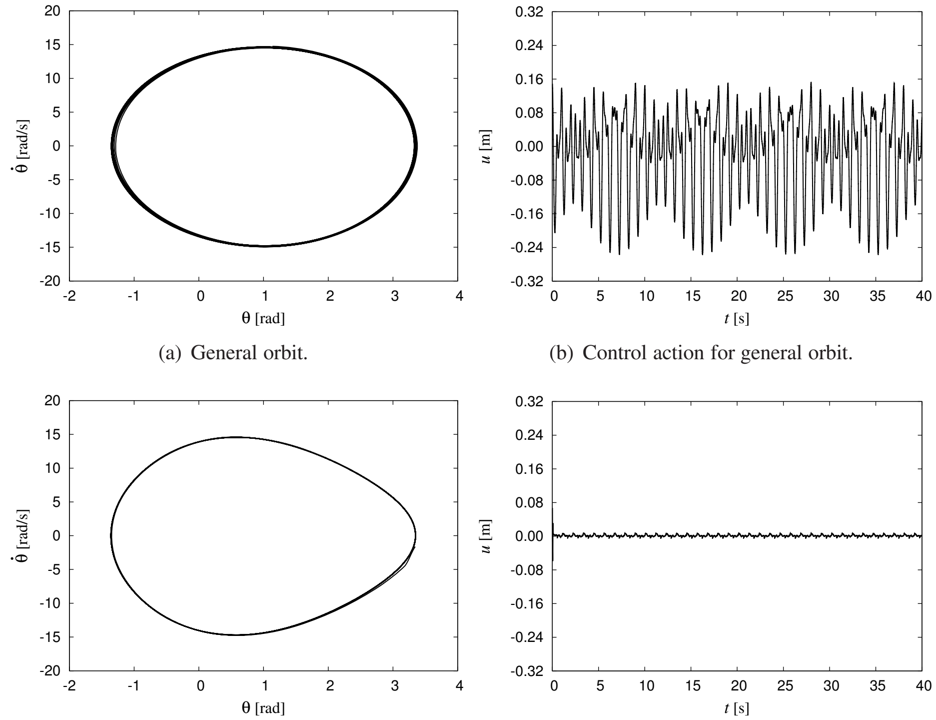


Fig. 2 Adopted fuzzy membership functions.

considered. Moreover, the dry friction is neglected being treated as unmodeled dynamics. Under this assumption, it is not taken into account within the design of the control law. On this basis, it is assumed estimation values as $\hat{\zeta} = 1.9 \times 10^{-5}$ kg m^2/s and $\hat{\mu} = 0$. The other estimates in both $\hat{f}$ and $\hat{h}$ are chosen based on the assumption that model coefficients are perfectly known. The other used parameters are $\mathcal{F} = 1.25$; $\mathcal{D} = 7.0$; $\mathcal{H} = 1.0$; $\phi = 2.0$; $\lambda = 0.8$; $\eta = 0.05$ and $\vartheta = 0.2$.

Chaos control of the nonlinear pendulum is now performed. Figure 3 presents the tracking of a general artificial orbit and a period-1 UPO, showing the phase space and the related control action. Note that, even in the presence of both structured and unstructured uncertainties, the adaptive fuzzy sliding mode controller (AFSMC) is capable to stabilize both natural and general orbits. Although both orbits are similar, it should be highlighted that the controller requires less effort to stabilize the UPO. In fact, the control action u, Fig. 3(b) and Fig. 3(d), represents the length variation in the string and only tiny variations are required to provide such different dynamic behaviors, which actually allows a great flexibility for the controlled nonlinear system.



(a) General orbit. (b) Control action for general orbit.

(c) Period-1 UPO. (d) Control action for period-1 UPO.

Fig. 3 Tracking of a general orbit and a period-1 UPO.

Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on chaos control procedures. Noise reduction schemes for chaotic noisy time series [8, 14, 21, 29, 31–33] or Kalman filtering [22, 23] are alternatives to deal with this kind of situation, however, these subjects are not employed here. Reference [25] says that the efficacy of the OGY method is close related to the noise level. Reference [36] also studied the effect of noise in OGY method, confirming the previous conclusion. Reference [26] presented an analysis of noisy signals of the nonlinear pendulum using a semi-continuous method concluding that the increase of the number of control station can compensate the noise effect. Here, the effect of noise on the AFSMC method applied to a nonlinear pendulum is investigated verifying the influence on the system stabilization and on the required control action.

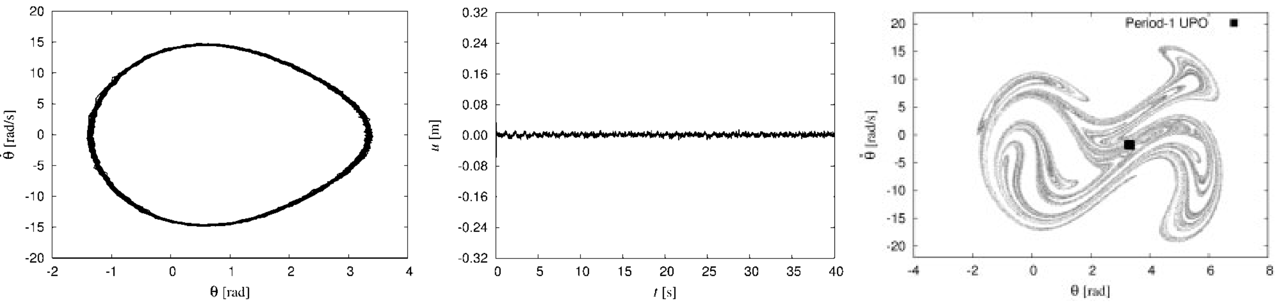
In order to simulate experimental noisy data sets, a white Gaussian noise is introduced in the signal:

$$\bar{\boldsymbol{\theta}}(t) = \boldsymbol{\theta}(t) + \boldsymbol{\epsilon}, \tag{8}$$

where $\bar{\boldsymbol{\theta}}$ represents the measured state vector, $\boldsymbol{\theta}$ states for the clean signals, and $\boldsymbol{\epsilon}$ the white Gaussian noise that is generated using the polar form of Box-Muller transformation [7]. The noise level is parametrized by the standard deviation of the clean signal (S_{signal}). Therefore, the standard deviation of the noise, S_{noise}, is a fraction γ of S_{signal}:

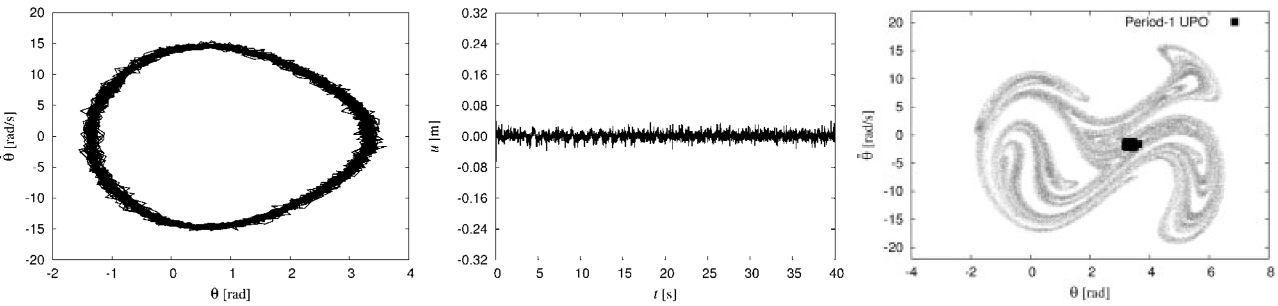
$$\gamma = \frac{S_{\text{noise}}}{S_{\text{signal}}} \times 100 \quad (\%). \tag{9}$$

At this moment, the controller is employed to stabilize the same period-1 UPO contaminated with three different noise levels, represented by different values of γ: 1%, 3% and 5%. Figures 4–6 show the stabilization of this period-1 UPO showing the phase space, the control action and the Poincaré section embedded in the related noisy strange attractor.



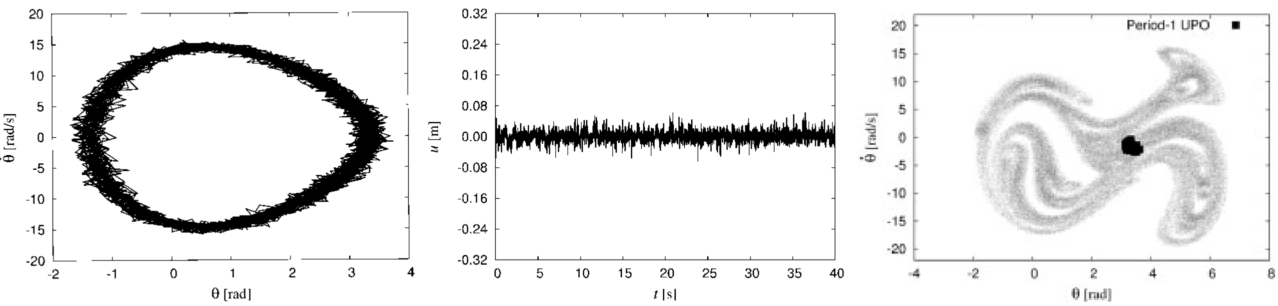
(a) Phase space. (b) Control action. (c) Poincaré section.

Fig. 4 Tracking of a period-1 UPO with $\gamma = 1\%$.



(a) Phase space. (b) Control action. (c) Poincaré section.

Fig. 5 Tracking of a period-1 UPO with $\gamma = 3\%$.

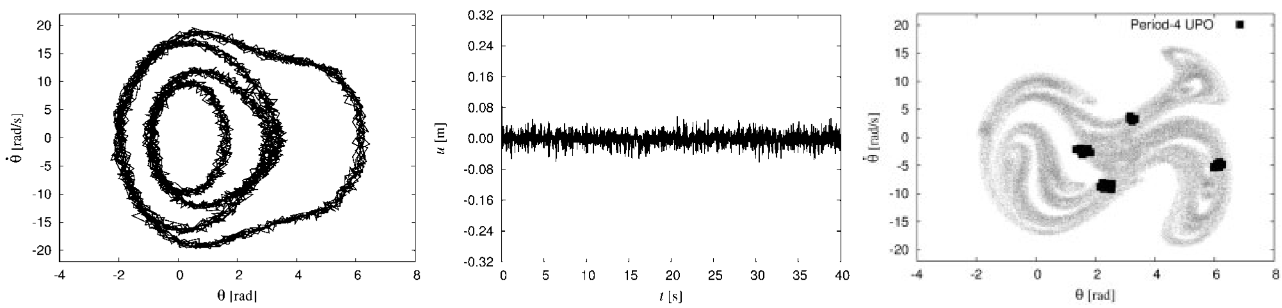


(a) Phase space. (b) Control action. (c) Poincaré section.

Fig. 6 Tracking of a period-1 UPO with $\gamma = 5\%$.

As observed in Figs. 4–6, the proposed control scheme allows the UPOs stabilization even when noisy signals are considered. Nevertheless, it can be verified that the increase of the noise amplitude causes a proportional increase of the control effort and a decrease in the tracking performance.

Finally, to ratify the ability of the AFSMC to track more intricate UPOs even in the presence of noisy signals, Fig. 7 presents the tracking of a period-4 UPO with $\gamma = 5\%$, showing the phase space, the control action and the Poincaré section as well. Once again, the controller is able to stabilize this UPO showing its capability to deal with noisy signals.



(a) Phase space. (b) Control action. (c) Poincaré section.

Fig. 7 Tracking of a period-4 UPO with $\gamma = 5\%$.

5 Conclusions

In this paper, an adaptive fuzzy sliding mode controller is merged with the close return method for the stabilization of UPOs in a chaotic pendulum. The adoption of the switching variable s in the premise of the fuzzy rules, instead of the state variables, leads to a smaller number of fuzzy sets and rules. By means of numerical simulations, the robustness of the AFSMC against modeling inaccuracies is evaluated considering structured and unstructured uncertainties. It could be verified that the proposed scheme is suitable not only for unstable periodic orbits but also for non-natural orbits. In fact, it should be emphasized that less effort is needed to stabilize an UPO when compared with a general orbit. Noisy signals are also investigated showing the controller capability to deal with this kind of uncertainty. In general, the adopted procedure is able to perform chaos control even in situations where high uncertainties are involved.

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References

[1] D. Auerbach, P. Cvitanović, J. P. Eckmann, G. Gunaratne, and I. Procaccia, Exploring chaotic motion through periodic orbits, Phys. Rev. Lett. **58**(23), 2387–2389 (1987).
[2] W. M. Bessa and R. S. S. Barrêto, Adaptive fuzzy sliding mode control of uncertain nonlinear systems, Controle & Automação **21**(2), 117–126 (2010).
[3] W. M. Bessa, A. S. De Paula, and M. A. Savi, Chaos control using an adaptive fuzzy sliding mode controller with application to a nonlinear pendulum, Chaos, Solitons, Fractals **42**(2), 784–791 (2009).
[4] W. M. Bessa, M. S. Dutra, and E. Kreuzer, Depth control of remotely operated underwater vehicles using an adaptive fuzzy sliding mode controller, Robot. Autonom. Syst. **56**(8), 670–677 (2008).
[5] W. M. Bessa, M. S. Dutra, and E. Kreuzer, An adaptive fuzzy sliding mode controller for remotely operated underwater vehicles, Robot. Autonom. Syst. **58**(1), 16–26 (2010).
[6] W. M. Bessa, M. S. Dutra, and E. Kreuzer, Sliding mode control with adaptive fuzzy dead-zone compensation of an electro-hydraulic servo-system, J. Intell. Robotic Syst. **58**(1), 3–16 (2010).
[7] G. E. P. Box and M. E. Muller, A note on the generation of random normal deviates, Ann. Math. Statist. **29**(2), 610–611 (1958).
[8] M. Davies, Noise reduction schemes for chaotic time series, Physica D **79**(2–4), 174–192 (1994).
[9] R. J. De Korte, J. C. Schouten, and C. M. V. Van den Bleek, Experimental control of a chaotic pendulum with unknown dynamics using delay coordinates, Phys. Rev. E **52**(4), 3358–3365 (1995).
[10] A. S. De Paula and M. A. Savi, A multiparameter chaos control method based on OGY approach, Chaos, Solitons, Fractals **40**(3), 1376–1390 (2009).
[11] A. S. De Paula and M. A. Savi, Comparative analysis of chaos control methods: A mechanical system case study, Int. J. Non-Linear Mech. **46**(8), 1076–1089 (2011).
[12] A. S. De Paula, M. A. Savi, and F. H. I. Pereira-Pinto, Chaos and transient chaos in an experimental nonlinear pendulum, J. Sound Vibr. **294**(3), 585–595 (2006).
[13] U. Dressler and G. Nitsche, Controlling chaos using time delay coordinates, Phys. Rev. Lett. **68**(1), 1–4 (1992).
[14] N. Enge, T. Buzug, and G. Pfister, Noise reduction on chaotic attractors, Phys. Lett. A **175**(3–4), 178–186 (1993).
[15] L. F. P. Franca and M. A. Savi, Distinguishing periodic and chaotic time series obtained from an experimental nonlinear pendulum, Nonlinear Dyn. **26**(3), 253–271 (2001).

[16] C. C. Fuh and P. C. Tung, Robust control for a class of nonlinear oscillators with chaotic attractors, Phys. Lett. A **218**(3–6), 240–248 (1996).
[17] B. Hübinger, R. Doerner, W. Martienssen, M. Herdering, R. Pitka, and U. Dressler, Controlling chaos experimentally in systems exhibiting large effective Lyapunov exponents, Phys. Rev. E **50**(2), 932–948 (1994).
[18] J. S. R. Jang, C. T. Sun, and E. Mizutani, Neuro Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence (Prentice Hall, New Jersey, 1997).
[19] K. Konishi, M. Hirai, and H. Kokame, Sliding mode control for a class of chaotic systems, Phys. Lett. A **245**(6), 511–517 (1998).
[20] B. Kosko, Fuzzy systems as universal approximators, IEEE Transactions on Computers **43**(11), 1329–1333 (1994).
[21] E. J. Kostelich and T. Schreiber, Noise reduction in chaotic time-series data: A survey of common methods, Phys. Rev. E **48**(3), 1752–1763 (1993).
[22] T. Lefebvre, H. Bruyninckx, and J. D. Schutter, Kalman Filters for Nonlinear Systems: A Comparison of Performance, Tech. Rep. 01R033 (Department of Mechanical Engineering, Katholieke Universiteit Leuven, Belgium, 2001).
[23] H. Leung, Z. Zhu, and Z. Ding, An aperiodic phenomenon of the extended Kalman filter in filtering noisy chaotic signals, IEEE Trans. Signal Process. **48**(6), 1807–1810 (2000).
[24] M. Otani and A. J. Jones, Guiding Chaotic Orbits, Tech. rep. (Imperial College of Science Technology and Medicine, London, 1997).
[25] E. Ott, C. Grebogi, and J. A. Yorke, Controlling chaos, Phys. Rev. Lett. **64**(11), 1196–1199 (1990).
[26] F. H. I. Pereira-Pinto, A. M. Ferreira, and M. A. Savi, Chaos control in a nonlinear pendulum using a semi-continuous method, Chaos, Solitons, Fractals **22**(3), 653–668 (2004).
[27] K. Pyragas, Continuous control of chaos by self-controlling feedback, Phys. Lett. A **170**(6), 421–428 (1992).
[28] K. Pyragas, Delayed feedback control of chaos, Philos. Trans. R. Soc. A **364**(1846), 2309–2334 (2006).
[29] T. Sauer, A noise reduction method for signals from nonlinear systems, Physica D **58**(1–4), 193–201 (1992).
[30] M. A. Savi, F. H. I. Pereira-Pinto, and A. M. Ferreira, Chaos control in mechanical systems, Shock Vibr. **13**(4–5), 301–314 (2006).
[31] T. Schreiber and P. Grassberger, A simple noise-reduction method for real data, Phys. Lett. A **160**(5), 411–418 (1991).
[32] T. Schreiber and M. Richter, Fast nonlinear projective filtering in a data stream, Int. J. Bifurcation Chaos **9**(10), 2039–2045 (1999).
[33] K. Shin, J. K. Hammond, and P. R. White, Iterative SVD method for noise reduction of low-dimensionals chaotic time-series, Mech. Syst. Signal Processing **13**(1), 115–124 (1999).
[34] J. J. E. Slotine, Sliding controller design for nonlinear systems, Int. J. Control **40**(2), 421–434 (1984).
[35] P. So and E. Ott, Controlling chaos using time delay coordinates via stabilization of periodic orbits, Phys. Rev. E **51**(4), 2955–2962 (1995).
[36] M. L. Spano, W. L. Ditto, and S. N. Rauseo, Exploitation of chaos for active control: An experiment, J. Intell. Mater. Syst. Struct. **2**(4), 482–493 (1990).
[37] H. H. Tsai, C. C. Fuh, and C. N. Chang, A robust controller for chaotic systems under external excitation, Chaos, Solitons, Fractals **14**(4), 627–632 (2002).
[38] X. Yu, Variable structure control approach for controlling chaos, Chaos, Solitons, Fractals **8**(9), 1577–1586 (1997).